VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD B.E. (CSE: CBCS) III-Semester Main Examinations, December-2018

## Discrete Structures

Time: 3 hours
Max. Marks: 60
Note: Answer ALL questions in Part-A and any FIVE from Part-B

| Q. No | Stem of the Question | M | L | CO | PO |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 2. | Define principle of duality and Write the dual of $\neg \mathrm{p} \vee \neg \mathrm{q} \wedge \mathrm{T}_{0}$ | 2 | 2 | 1 | 1 |
| 3. | Compute the prime factors of 540, 504 | 2 | 2 | 2 | 1,2 |
|  | What is partition? Compute the partitions for set $\mathrm{A}=\{2,3,5,6,7,1,9\}$ | 2 | 2 | 2 | 1 |
| 5. | Compute the recurrence relation for the number of comparisons to sort $n$ numbers using bubble sort | 2 | 2 | 3 | 1 |
|  | Compute the generating function for $\mathrm{P}_{\mathrm{d}}(\mathrm{n})$ where $\mathrm{P}_{\mathrm{d}}$ is number of partitions of positive integer n into distinct Summands | 2 | 3 | 3 | 1 |
| 7. | Define commutative Ring | 2 | 2 | 4 | 1,2 |
| 8. | What is Euler's Totient function? and calculate Euler's Totient of 35 | 2 | 3 | 4 | 1,2 |
| 9. | $\mathrm{G}=\{0,1,2,3,4,5\}$ be a group under addition modulo 6 . Find the inverse of 5 | 2 | 2 | 5 | 1,2 |
| $10 .$ | What is the minimum distance of a code consisting of the code words 001010,011100,010111,011110,101001? | 2 | 4 | 5 | 1,2 |
| Part-B ( $5 \times 8=40 \mathrm{Marks}$ ) |  |  |  |  |  |
| 11. a | Justify $\neg \mathrm{p}$ is a valid conclusion from $\mathrm{p} \rightarrow \mathrm{r}, \mathrm{r} \rightarrow \mathrm{s}, \mathrm{t} \vee \neg \mathrm{s}, \neg \mathrm{t} \vee \mathrm{u}, \neg \mathrm{u}$ | 4 | 2 | 1 | 1 |
|  | Find $\operatorname{gcd}(\mathrm{a}, \mathrm{b})$ if $\mathrm{a}=1820$ and $\mathrm{b}=231$ using Euclidian algorithm | 4 | 2 | 1 | 1 |
| 12. a) | Let $\mathrm{A}=\{1,2,3,4\}, \mathrm{b}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ and $\mathrm{c}=\{\mathrm{w}, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$ with $\mathrm{f}: \mathrm{A}->\mathrm{B}$ and $\mathrm{g}: \mathrm{B}->\mathrm{C}$ given by $\mathrm{f}=\{(1, \mathrm{a}),(2, \mathrm{a}),(3, \mathrm{~b}),(4, \mathrm{c})$ and $\mathrm{g}=\{(\mathrm{a}, \mathrm{x}),(\mathrm{b}, \mathrm{y}),(\mathrm{c}, \mathrm{z})\}$ for each element of A. Show that $(g \circ f)^{-1}=f^{l} \circ g^{-1}$ | 4 | 3 | 2 | 1 |
|  | In how many ways can the 26 letters of the alphabet be permuted so that non of the patterns car, dog, pun occurs? | 4 | 2 | 2 | 1 |
| 13. a) | Solve the recurrence relation $2 a_{n}=7 a_{n-1}-3 a_{n-2}, n \geq 2$ and $a_{0}=2, a_{1}=5$ | 4 | 3 | 3 | 1,2 |
|  | Find coefficient of $x^{8}$ in the series $\frac{1}{(1-2 x)^{2}(1-3 x)}$ | 4 | 2 | 3 | 1,2 |
|  | Find the minimum value of $X$ which satisfies the following simultaneous equations $\begin{aligned} & X \equiv 14(\bmod 31) \\ & X \equiv 16(\bmod 32) \\ & X \equiv 18(\bmod 33) \end{aligned}$ | 4 | 4 | 4 | 1,2 |
|  | Compute $342{ }^{15} \mathrm{mod} 61$ using Euler's theorem | 4 | 3 | 4 | 1 |

15. a) Verify $\mathrm{G}=\{0,1,2,3,4,5,6,7\}$ is a group under addition modulo 8 .
b) Prove that the set of idempotent elements of M for any abeleian monoid ( $\mathrm{M},{ }^{*}$ ) forms a sub monoid
16. a) Apply Mathematical induction to verify $\sum_{i=1}{ }^{n} i\left(2^{i}\right)=2+(n-1) 2^{n+1}$
b) Find prime factorization of 327236910
17. Answer any two of the following:
a) There are four colors poker chips -red, white, green and blue. Find and solve the recurrence relation for the number of ways to stack $n$ of these poker chips so that there are no consecutive blue chips.
b) Show that $\left(\mathrm{M},{ }^{*}\right)$ is an abelian group where $M=\left\{A, A^{2}, A^{3}, A^{4}\right\}$ where $A=\left[\begin{array}{rr}0 & 1 \\ -1 & 0\end{array}\right]$
And * is an ordinary matrix multiplication, Also prove that ( $\mathrm{M},$.$) is$ isomorphic to the abelian group $(G, X) G=\{1,-1, i,-i\}$ where $X$ is an ordinary multiplication
c) Define the encoding function $\mathrm{E}: \mathrm{Z}_{2}{ }^{3}{ }^{->} \mathrm{Z}_{2}{ }^{6}$ by means of the parity-check matrix H given as below
a) determine all code words
b) Does this code correct all single errors in transmission

$$
H=\left(\begin{array}{lllll}
1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1
\end{array}\right)
$$

| 4 | 2 | 5 | 1 |
| :---: | :---: | :---: | :---: |
| 4 | 3 | 5 | 1,2 |


| 4 | 2 | 1 | 1 |
| :--- | :--- | :--- | :--- |

$\begin{array}{llll}4 & 2 & 2 & 1\end{array}$
$4 \quad 2 \quad 31,2$

4241,2

4351,2

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

| S. No. | Criteria for questions | Percentage |
| :---: | :--- | :---: |
| 1 | Fundamental knowledge (Level-1 \& 2) | 60 |
| 2 | Knowledge on application and analysis (Level-3 \& 4) | 40 |
| 3 | *Critical thinking and ability to design (Level-5 \& 6) <br> (*wherever applicable) | --- |

